Role of Porosity in Filtration: XII. Filtration with Sedimentation

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Filtration on horizontal surfaces facing upward is accompanied by sedimentation. Material balances that are based solely on the volume of filtrate and neglect sedimentation flux lead to an understatement of the solids deposited in the cake and potentially large errors in calculated values of the average specific resistance α_{av}. In a gravitational sedimentation experiment with kaolin, the value of α_{av} neglecting sedimentation was 3.75 times greater than the value including the effect of sedimentation. In addition to errors due to neglect of sedimentation, CATSCAN studies show that the slurry concentration above the cake increases with time, contrary to usual assumptions. In a manner similar to batch sedimentation in a closed cylinder, characteristics of constant composition arose from the cake surface. Approximate predictions based on a combination of traditional sedimentation and filtration theory were in accord with the CATSCAN data. Existing filtration theory must be substantially modified to account for the effect of sedimentation.

Introduction

Conventional filtration theory is based on a two-resistance (R_c = cake, R_m = supporting medium) model in the form

$$q = \frac{dv}{dt} = \frac{p}{\mu(R_c + R_m)} = \frac{p}{\mu(\alpha_{av}\omega_c + R_m)}, \tag{1}$$

where q = instantaneous filtrate flow rate, v = filtrate volume/unit area, p = applied pressure, and μ = viscosity. The cake resistance is given by the product of the average specific resistance α_{av} and the volume of inert cake solids/unit area ω_c. Although the usual definition encountered in the literature employs w_c, mass of solids/unit area, there are distinct advantages accruing to the use of volumetric units in simplicity of formulas, correlations, and predictive power. Although Eq. 1 has been given exalted status in the past as the "fundamental" formula governing filtration, it has actually been the subject of much criticism and is based on a number of faulty assumptions. In spite of deficiencies, Eq. 1 is very useful in analysis when viewed as an empirical approximation. It provides an instantaneous picture of the relationship among the variables, which include flow rate q, applied pump pressure p, cake resistance, $R_c = \alpha_{av}\omega_c$, and medium resistance R_m. Integration of Eq. 1 to obtain filtrate volume as a function of time t requires:

1. Specification of pump characteristics, that is, p as a function of q.
2. Relation beween v and ω_c as obtained from a volumetric balance.
3. Constitutive equations relating the average values of the specific resistance, permeability, and volume fraction of solids in the cake ϵ_{sav} to the pressure drop Δp_c across the cake.

In this article, attention will be focused on constant applied pressure laboratory practice, thereby simplifying the specifications required in item 1. Most importantly, it is assumed that the local superficial flow rate q is constant throughout the cake and equal to dv/dt. Although it is known that q varies with distance (Sorensen, 1992), the variation is small for dilute slurries encountered in pressure filtration.

Determination of parameters in empirical constitutive relations is accomplished by fitting theoretical formulas with experimental data. Conventional techniques recommended for analysis of data obtained from filtration experiments and the theoretical formulas on which the methodology depends have frequently been faulty. This article will address deficiencies related to laboratory experiments involving constant pressure filtration on horizontal surfaces. As sedimentation effects can produce errors of several orders of magnitude with dilute slurries filtered at low pressure, published values (including the authors') should be viewed with caution. Sperry (1917), in the second of his articles in which he introduced a formula equivalent to Eq. 1, pointed to sedimentation as a possible cause of deviation from his predicted parabolic relationship between v and t. Somewhat later, Ruth (1952, 1992) presented data showing large differences in deposition on horizontal surfaces facing upward and downward. More recently, Christensen and Dick (1985) and Bockstal et al. (1985) have addressed the problem of sedimentation.

In order to integrate Eq. 1 to obtain v as a function of t, it is necessary to relate ω_c to v. Customarily, a material balance is written in which the volume of slurry is equated to the sum of the cake and filtrate volumes. Neglect of sedimentation leads to a potentially large underestimate of the volume of solids in the cake.

Sedimentation

As long as a density difference exists in an unstirred suspension, the solids will settle under gravity. If liquid is moving down, the solids will settle at a higher velocity. If the liquid is moving up, the solids may move down, be stationary as in a fluidized bed, or move up at a slower velocity than the liquid. In horizontal filters (leaf, automatic press, belt presses, Nutsch), liquid flows down, and the solids settle faster than the liquid. In upflow as in a continuous rotary drum filter, the liquid is sucked up by vacuum through the drum, and agitators are required to offset the effect of sedimentation.

The nature of sedimentation is closely tied to concentration. In Figure 1A, the relative settling velocity $u_{sR} = u_s - u_L (u_s, u_L$ = velocities of solid and liquid) is shown as a function of concentration. In the very dilute range, particles settle independently. As concentration ϕ_s increases and settling of large particles is impeded by the presence of small particles, a point is reached where all particles presumably have identical velocities and settle as a "zone." In contrast to the idealized picture of Figure 1, no sharp line of demarcation exists between the diffuse and zone settling regimes. Ultimately, a point is reached where the solids form a cake capable of transmitting stresses through points of contact. The solids then enter into a compression zone where compaction depends upon accumulated stress in the particulate matrix. As a crude approximation, we assume that the null stress solidosity ϵ_{s0} (volume fraction of solids under zero compactive stress) marks the beginning of the cake zone. The value of ϵ_{s0} varies with the slurry concentration (Tiller and Khatib, 1984).

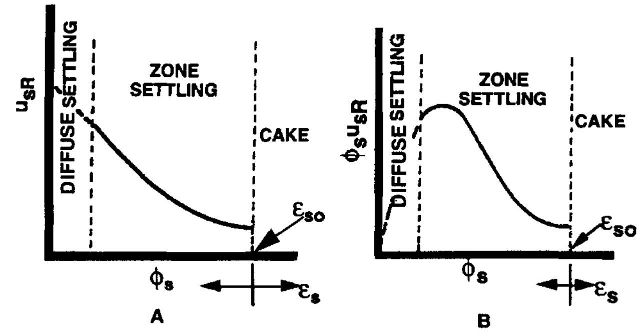

Figure 1. Relative settling velocities and relative flux as a function of concentration.

The assumption of zone settling primarily depends upon visual observation of the top particle/liquid interface in countless sedimentation tests. Most investigations have been based on slurries containing dispersed, mono-sized particles well out of the colloidal range. Such suspensions tend to behave in the classic zone settling mode. As the size range stretches out and particles form flocs, the supposedly sharply defined surface presumed to exist as a moving shock in zone settling is apt to be replaced by a blurry surface in which there is no well-defined jump in concentration. The rate of cake buildup depends on the flux of solids at the cake surface. Bulk movement due to filtrate flow and the relative sedimentation flux $\phi_s u_{sR}$ (rather than relative velocity) account for solid deposition. As can be seen in Figure 1B, the relative sedimenting flux reaches a maximum and then decreases. The concentration at which the flux maximizes can be estimated if an analytical equation relating u_{sR} to ϕ_s is available (Richardson and Zaki, 1954; and Scott, 1984). The well-known Richardson-Zaki equations apply principally to discrete, uniform, spherical particles for which ϵ_{s0} is in the neighborhood of 0.6–0.65. Formulas for flocculated slurries where ϵ_{s0} takes on values that may be as low as 0.1 or less have received less attention, and *a priori* prediction of sedimentation velocities is in general not possible. Since many slurries destined for pressure filters tend to be "dilute" (the ratio ϕ_s/ϵ_{s0} rather than the magnitude of ϕ_s determines the degree of diluteness), settling flux is expected to be a relatively large fraction of the solid flux. When concentrated slurries ($\phi_s/\epsilon_{s0} > 0.6$ as a crude guideline) are involved in filtration, settling rates decrease in proportion to the total solid flux, and settling is of less importance.

Volumetric Balance

Attention in this article will be focused on deposition of particles in the zone settling region on a horizontal filter surface with the suspension located above the surface and flow being vertically downward (Christensen and Dick, 1985). In the simplest experiments, the entire charge of slurry is introduced into the vessel at the start; no feed is added; and there is no mixing. For the majority of cases in which the suspended solids possess a larger density than the liquid, the velocity u_s of the solids approaching the cake surface will be larger than the liquid velocity u_L and will be given by

$$u_s = u_L + u_{sR}. \tag{2}$$

As long as $u_{sR} > 0$, the velocity of the solids will be greater than the velocity of the liquid at the cake surface. Since conventional theory assumes $u_{sR} = 0$, the rate of deposition of solids on a horizontal medium will be underestimated; and errors will be committed in relating v filtrate volume/unit

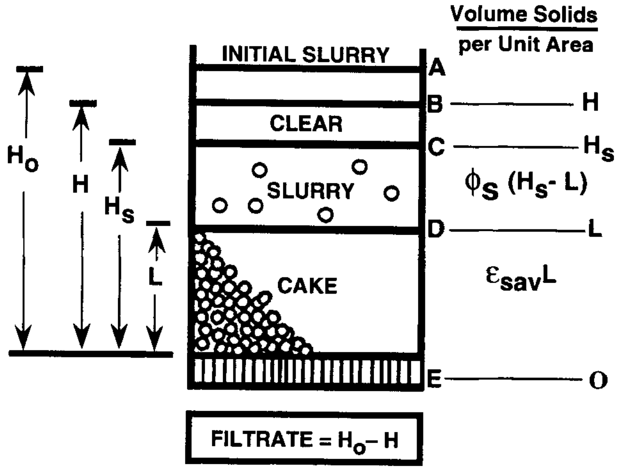

Figure 2. Quantities involved in filtration with sedimentation.

area to ω_c, the volume of solids/unit area. The relative velocity u_{sR} is generally assumed to be a unique function of the suspension volumetric solid fraction ϕ_s. Problems associated with sedimentation are particularly acute in laboratory filters, Buchner funnels, and capillary suction apparatus.

Filtration with Sedimentation

A schematic view of a cell with a horizontal filter surface is shown in Figure 2. Slurry is introduced into the filter, and pressurized gas is introduced above the top surface. With an original slurry height of H_0, the liquid level drops to H while the solids sediment to point H_s. The supernatant region $H - H_s$ (BC) is free of solids. The region represented by $H_0 - H$ equals the filtrate volume/unit area. The volume fraction of solids in the slurry is assumed to be constant in the slurry region given by $H_s - L$ (CD).

The solidosity (volume fraction of solids) ϵ_s varies with distance. The maximum value of ϵ_s occurs at the point E where the cake rests on the supporting medium. It decreases to its minimum at the cake surface D where it equals ϵ_{s0}. The total volume of solids per unit area originally present, $\phi_s H_0$, equals the sum of the volume per unit area in (1) the slurry, $\phi_s(H_s - L)$; and (2) the cake, $\epsilon_{sav}L$, where ϵ_{sav} = average cake solidosity.

The suspension concentration ϕ_s above the cake will remain constant if no constant concentration characteristics of the Kynch type arise (Kynch, 1952; Tiller, 1981; Font, 1988). Experimental data involving visual observation of the surfaces C and D in Figure 2 for a dilute kaolin suspension, however, point to the appearance of characteristics arising from the cake surface. Determination of concentrations by X-ray measurements in a computerized axial tomographic scanner (CATSCAN) confirmed the presence of changing slurry concentrations typical of rising characteristics. In addition, the CATSCAN data showed that no sharp concentration differences existed at either the supernatant/slurry or the slurry/cake interfaces; and, thus, idealized zone settling did not exist. In the derivations that follow, it will be assumed that ϕ_s remains constant and that zone settling exists. Although approximate, the resulting formulas will provide reasonable predictive power for a large range of conditions.

Use of Eq. 1 requires that the volume/unit area ω_c of solids be related to the filtrate volume/unit area v. Two different formulas result depending on whether or not the cake thickness is measured as a function of time. Equating the total volume of solids to the volume of solids in the slurry plus the volume of solids in the cake:

$$\phi_s H_0 = \phi_s(H_s - L) + \epsilon_{sav} L. \tag{3}$$

The solids in the cake are given by

$$\omega_c = \int_0^L \epsilon_s \, dx = \epsilon_{sav} L. \tag{4}$$

The height of the interface H_s is related to H by

$$H_s = H - u_{sR} t. \tag{5}$$

Substituting Eq. 5 into Eq. 3, rearranging, and noting that $v = H_0 - H$ leads to

$$\phi_s(H_0 - H) = \phi_s v = (\epsilon_{sav} - \phi_s)L - \phi_s u_{sR} t. \tag{6}$$

The quantity ω_c that appears in Eq. 1 can be obtained from Eqs. 2 and 6:

$$\omega_c = \epsilon_{sav} L = \phi_s(v + L) + \phi_s u_{sR} t = \phi_s(H_0 - H_s + L). \tag{7}$$

The quantity $(v + L)$ is the sum of filtrate and cake volumes/unit area. When no sedimentation is present, $(v + L)$ equals the slurry volume that has been transformed into cake and filtrate. The u_{sR} term in Eq. 7 represents the volume of solids/unit area that has been *transported through the constant composition slurry and into the cake*. The use of Eq. 7 requires determination of L as a function of t. The last term in Eq. 7 permits calculation of ω_c from directly measurable quantities.

Another form for ω_c can be obtained by solving for L in Eq. 6 and multiplying by ϵ_{sav} to produce

$$\omega_c = \frac{\phi_s}{1 - \phi_s/\epsilon_{sav}}(v + u_{sR} t) = c(v + u_{sR} t), \tag{8}$$

where $c = \phi_s \epsilon_{sav}/(\epsilon_{sav} - \phi_s)$. When L is not measured experimentally, Eq. 8 can be employed to calculate ω_c. The settling velocity can be obtained in a separate experiment. The value of ϵ_{sav} can be determined by measuring the solid concentration of the cake at the termination of filtration. Although ϵ_{sav} varies as the pressure drop across the cake increases with time, the variation is usually small. If ϕ_s/ϵ_{sav} is less than 0.1, the effect of the variation will normally be of little importance.

A useful formula for calculating the average cake solidosity results from solving Eq. 7 for ϵ_{sav} as follows

$$\epsilon_{sav} = \phi_s\left(1 + \frac{v}{L} + \frac{u_{sR}t}{L}\right) = \phi_s(H_0 - H_s + L)/L. \quad (9)$$

If L and H_s are determined experimentally, ϵ_{sav} can be estimated as a function of time. Substitution of ω_c from Eq. 8 into Eq. 1 leads to an integrable equation if ϵ_{sav}, α_{av}, and u_{sR} are assumed to be constant. The use of ω_c from Eq. 7 with Eq. 1 does not lead to an integrable form, as the variable L is then present in addition to v and t. As discussed later, however, combining Eqs. 1 and 7 leads to a superior procedure for data analysis.

Pressure filtration

Substituting Eq. 8 into Eq. 1 and rearranging yields a first-order linear differential equation

$$\frac{dt}{dv} - Gu_{sR}t = Gv + \frac{\mu R_m}{p} \quad (10)$$

where $G = \mu c \alpha_{av}/p$. Equation 10 was first proposed by Bockstal et al. (1985). To obtain an analytical solution of Eq. 10 requires that G be constant, that is, c, α_{av}, and u_{sR} must be constant. As the pressure drop across the cake varies, both c and α_{av} will vary. In general, variations diminish rapidly with time; and the integral of Eq. 10 would be expected to portray experimental data with reasonable accuracy after the first few minutes of a constant pressure filtration. Integrating Eq. 10 and rearranging yields

$$c\alpha_{av}(v + u_{sR}t) = \left(\frac{p}{\mu u_{sR}} + R_m\right)(e^{Gu_{sR}v} - 1), \quad (11)$$

which is equivalent to the solution given by Bockstal et al. (1985). Expanding the exponential to the cubic in v and rearranging leads to

$$\frac{pt}{\mu v} = \frac{\alpha_{av}}{2}cv + R_m + u_{sR}\frac{\mu\alpha_{av}cv}{2p}\left[\frac{\alpha_{av}cv}{3} + R_m + \frac{\mu\alpha_{av}cvu_{sR}}{3p}\right]. \quad (12)$$

If $u_{sR} = 0$, Eq. 12 reduces to the usual parabolic form employed for analyzing data.

Equations 11 and 12 are valid up to the time that the solids in the suspension disappear, which occurs when $H_s = L$. Conditions existing at the point where all the solids have been deposited are represented by

$$\omega_c = \epsilon_{sav}L_f = \phi_s H_0 = \phi_s(v_f + L_f + u_{sR}t_f), \quad (13)$$

where subscript f denotes the values of L, v, and t at the point where all of the solids have been deposited. After point f is reached, pure liquid flows through the cake. Assuming that there is no cake compaction, the remaining liquid will flow through the cake at a constant rate given by Eq. 1 with $\omega_c = cv_f$. Using a value of the medium resistance determined during the cake-formation period, the average specific resistance during the pure liquid flowthrough period can be obtained by rearranging Eq. 1 into

$$\alpha_{av}\omega_c = \alpha_{av}\phi_s H_0 = \frac{p}{\mu q_f} - R_m \quad (14)$$

where q_f represents the flow rate after all of the solids have been deposited. Flow through a bed having a constant mass of solid material provides a simple procedure for determining α_{av}. If the bed thickness is known, the average solidosity ϵ_{sav} and the average permeability can also be obtained.

Gravitational filtration

Cake filtration employing gravity as a driving force is not generally employed in industrial processes for materials having a low permeability. In the laboratory, gravitational filtration is useful for obtaining cake resistance at low pressure. The effect of sedimentation is proportionately larger in gravitational as compared with pressure filtration. Although rising characteristics are expected to be a significant factor, they will be neglected in the approximate theory to be developed.

In a falling head filtration, the pressure term in Eq. 10 is not constant and must be related to the height H of Figure 2. At $t = 0$, the pressure is given by

$$p_0 = g(\phi_s\rho_s + \phi_L\rho_L)H = g\rho_{sL}H \quad (15)$$

where ρ_{sL} = slurry density. At an arbitrary time, the hydrostatic pressure is determined by the clear liquid present in the pores of the cake, the no-solids region $(H - H_s)$, and the slurry in the region $(H_s - L)$. With dilute slurries used in this investigation, the pressure will be approximated by assuming that the pressure is given by $\rho_L gH$.

The filtrate volume $v = H_0 - H$ is shown in Figure 2. Replacing dv by $-dH$ and substituting for p in Eq. 10 leads to

$$\frac{dt}{dH} + G_1 u_{sR}\frac{t}{H} = -\left(G_1 H_0 + \frac{\mu R_m}{\rho_L g}\right)\frac{1}{H} + G_1 \quad (16)$$

where $G_1 = \mu c\alpha_{av}/\rho_L g$. The solution to this first-order linear equation subject to $H = H_0$ when $t = 0$ is

$$t = \left[\frac{\rho_L g H_0/u_{sR}}{\mu c\alpha_{av}u_{sR} + \rho_L g} + \frac{R_m}{c\alpha_{av}u_{sR}}\right]\left(\frac{H_0}{H}\right)^m + \frac{\mu c\alpha_{av}}{\mu c\alpha_{av}u_{sR} + 1}H + \frac{\alpha_{av}cH_0 + R_m}{\alpha_{av}cu_{sR}} \quad (17)$$

where $m = \mu c\alpha_{av}u_{sR}/\rho_L g$. Both the Bockstal formula for pressure filtration and Eq. 17 for gravitational operation with a falling head provide reasonable approximations where sedimentation effects are important. Both are subject to the upward moving concentration waves, however, which result when particles are deposited on the cake surface. When solids are deposited, they displace liquid that then tends to move upward. On the one hand, the characteristic concentration wave has a velocity with a direction counter to gravity; and on the other, the filtrate has an offsetting velocity in the downward direction. Depending upon the relative values of the characteristic and filtration velocities, the wave may escape and pass upward; or it may be buried in the cake. If the medium resistance is high, the filtration rate will be low, and sedimentation effects may predominate. Data to be presented later indicate that the characteristic wave is initially buried and then later escapes the cake as the filtrate velocity rapidly decreases with cake buildup. The liquid velocity slowdown caused by the particles impacting on the cake surface results in an increase in concentration of the slurry above the cake. The previous derivations that have rested on the assumption of constant slurry concentration will be impacted. The magnitude of the problem will be discussed in the experimental section of this manuscript.

Analysis of Experimental Data

In the analysis of filtration data, it is essential to differentiate approximations that enter the basic theory and experimental problems involving suspensions and cakes. In solid/liquid separation, it is customary to start theoretical analyses with some form of Darcy's empirical law, differential and overall mass or volume balances, and stress balances resulting from neglect of inertial forces in momentum balances. Constitutive relationships among local values of permeability, solidosity (volume fraction of solids), and p_s the Terzhagi effective solid stress are also necessary. The resulting partial differential equations (Risbud, 1974; Wakeman, 1978; Willis and Tosun, 1980; Tiller, 1981; Chen, 1987; Jonsson, 1992) have seldom if ever been tested directly. In most theories in the literature, it has been assumed that the velocity of the solids is zero even in compactible cakes, thereby violating the differential material balance. In spite of the approximate nature of equations based upon negligible solid velocity and constant superficial liquid velocity, q, the resulting formulas play a useful role in evaluating filtration operations.

Problems originate in the complex fluid dynamical and interfacial phenomena involving particles, aggregates, and the surrounding fluid. Realistic slurries undergoing transient behavior of interacting species defy precise characterization. Migration and deposition of colloidal particles, transient flocculation reactions, and hydraulic flow patterns in filters are among phenomena seldom considered. The overall or average values of permeability, solidosity, and specific resistance obtained from experimental data are subject to much uncertainty. It is against this background that the analysis of experimental data provided in this article must be judged.

Depending upon the experimental design, there are four measurable variables, v, t, L, and u_{sR}, appearing in the previous equations. In most experiments reported in the literature, only v and t are determined along with the final value of ϵ_{sav} obtained at the termination of the experiment. If u_{sR} and L are measured, additional information can be extracted from the data. It is possible to use either the differential equation or the integrated form in the analysis of experimental data. Use of the differential equation offers the distinct advantage of not requiring that α_{av} and ϵ_{sav} and, as a consequence, that c and G be constant. Potentially more information can be obtained from direct use of the differential equation, particularly when the thickness, L, is included as part of the data.

Equation 1 can be combined with Eq. 7 to give

$$\frac{p}{\mu q} = \alpha_{av}\phi_s(v + L + u_{sR}t) + R_m \tag{18}$$

where $q = dv/dt$. Combining Eq. 1 with Eq. 8 leads to

$$\frac{p}{\mu q} = \alpha_{av}c(v + u_{sR}t) + R_m. \tag{19}$$

Both Eqs. 18 and 19 require knowledge of u_{sR}. The cake thickness L must be determined if Eq. 18 is employed. Use of c as dictated by Eq. 19 depends upon finding ϵ_{sav} of the cake at the end of filtration.

Experimental methods for determining u_{sR} include:

1. Batch settling tests yielding results as shown in Figure 1 combined with filtrate volume vs. time data.
2. Observation of $(H - H_s)$ as a function of time: a. visual observation; b. absorption of radiation, gamma rays, or X-rays in a computerized axial tomographic scanner (CAT-SCAN).

Method 1. If batch settling tests are employed, the measured settling velocity u_s must be converted to a relative velocity. As the solids move down, liquid moves up in accord with

$$u_s\phi_s + u_L\phi_L = 0, \tag{20}$$

where u_L and ϕ_L are, respectively, the velocity and volume fraction of the liquid. The relative velocity is given by

$$u_{sR} = u_s - u_L = u_s/\phi_L = u_s/(1-\phi_s). \tag{21}$$

For dilute slurries where ϕ_L is close to unity, there is little difference between u_s and u_{sR}.

Method 2. If a slurry is filtered in a vessel with transparent walls, it is possible to observe the upper level H (Figure 2) of the supernatant liquid, the interface of the zone settling region H_s, and the cake thickness L as functions of time. The relative settling velocity then can be calculated by means of Eq. 5.

Alternatively X-rays or gamma rays can be used to obtain concentration profiles as well as the position of interfaces. A pressure filter was placed in a CATSCAN, and concentration profiles were obtained at various time intervals. Two-dimensional scans produced concentrations as functions of both depth x and width. The concentrations in the direction orthogonal to x were averaged (Tiller et al., 1991).

Filtration with transparent walls

In Figure 3 data are shown for filtration of a 3% by volume kaolin slurry under constant pressure. After introducing the slurry into the filter, the gas pressure was adjusted to a constant value, and cake deposition on the horizontal surface ensued. The heights of four interfaces were determined visually:

	Symbol	Path	Interface
(1)	H	AFB	Liquid/Gas
(2)	H_s	AEC	Clear supernatant/slurry
(3)	L	ODC	Slurry/cake
(4)	L	CB	Supernatant/cake

The distance $H_0 - H$ equals the filtrate volume/unit area v. The rapid drop in H with t at the start of filtration (point A) indicates that the supporting medium (filter paper) had a relatively small resistance. As cake built up along path ODC (L vs. t) the cake resistance increased rapidly; and the rate decreased as indicated by the slope ($dH/dt = -dv/dt = -q$) of H vs. t along curve AF.

The solids settled simultaneously, and the slurry/clear supernatant interface traced out the path AEC (H_s vs. t). The distance $H - H_s$ represents the product of the relative settling velocity (assumed constant) and the time, that is, $u_{sR}t$. When H_s reached C, all of the solids had settled out, and cake formation was complete. Clear liquid then flowed through the cake in a washing mode.

Two runs with different values of H_0 (74 and 230 mm) are shown in Figure 3. The paths shown correspond to the data for $H_0 = 230$ mm. In order to quantitatively compare the two runs, we note that the relative settling velocity and the rate of cake buildup, L vs. t, are independent of the initial height H_0 when the suspension concentration ϕ_s, viscosity, and applied pressure are equal. Consequently, $H_0 - H$ representing the filtrate and $H - H_s$ the distance through which the solids have settled are the same for $H_0 = 74$ and 230 mm. The initial point representing $H_0 = 74$ (+ symbol) has been translated up so that it coincides with $H_0 = 230$ mm at A, and $H_0 - H$ and $H - H_s$ were plotted for the period up to 365 seconds at which time cake formation was complete.

Relative settling velocity

The distance $H - H_s$ was linear in t during the constant rate settling period, and relative settling velocities were

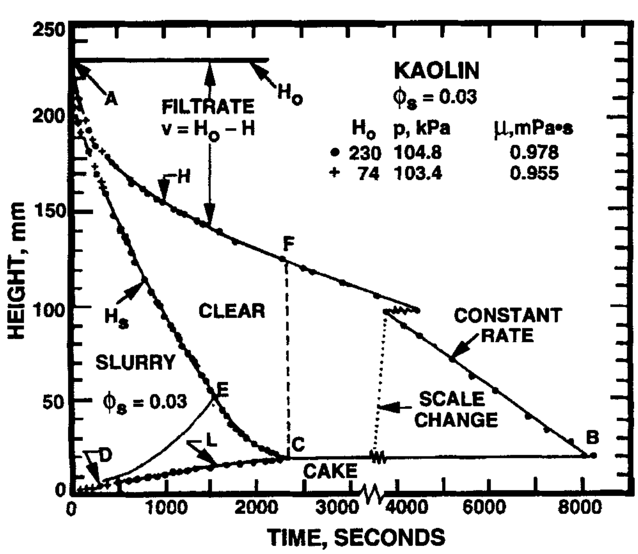

Figure 3. Filtration of kaolin slurries under a pressure of approximately one atmosphere.

$5.72E(-5)$ m/s for $H_0 = 230$ and $5.59\ E(-5)$ m/s for $H_0 = 74$. Correcting for viscosities leads to nearly identical values of u_{sR} for the two runs.

If the filtration were carried out in a filter with opaque walls, the H_s vs. t and L vs. t curves would not be available. Only v vs. t and the final value of the cake thickness could be measured. An estimate of the settling velocity could be obtained by locating F (Figure 3) as the point at which H vs. t becomes linear. Assuming the value of the final cake thickness (point B) was determined, the distance FC could be calculated and then divided by t_F to obtain u_{sR}. As will be shown, the value calculated by this procedure on a theoretical basis would be expected to be low. With $H_0 = 0.23$ m, $v_f = 0.1045$ m^3/m^2, $L_f = 0.0195$ m, and $t_f = 2{,}279$ s, $u_{sR} = (H_0 - v_f - L_f)/t_f = 4.65\ E(-5)$ m/s which is 83% of the value calculated from the plot of H_s vs. t in Figure 3. The corresponding calculation for the run with $H_0 = 74$ mm yielded $5.30E(-5)$ m/s, which is about 7% below the value found with the data of Figure 3.

Appearance of characteristics

In determining the relative sedimentation velocity for the kaolin ($H_0 = 230$ mm) shown in Figure 3, a deviation from linearity occurred at approximately $t = 1{,}650$ s in a plot of $(H - H_s)$ vs. t. The resulting decrease in velocity is typical of batch sedimentation experiments (Coulson et al., 1991). In a batch sedimentation process, solids move down while liquid is displaced upward. Sediment forms at the bottom where the solid velocity is reduced to zero. The sudden change in velocity at the bottom produces a signal that is transmitted upward through the suspension at constant velocity. The path of the signal is known mathematically as a characteristic. Theory indicates that the concentration of the suspension must be constant along a characteristic (Kynch, 1952). To illustrate the nature of characteristics, the data from Figure 3 have been converted into an equivalent batch sedimentation in Figure 4. The relative velocity u_{sR} obtained from Figure 3 must be changed to the absolute velocity as given by Eq. 20 for batch sedimentation. The observed velocity of the suspension/supernatant interface would then be $(1 - \phi_s)u_{sR}$. Since $\phi_s = 0.03$, the change is small. If the height of the upper surface remained at $H_0 = 230$ mm in a batch sedimentation, the slurry/supernatant interface would correspond to $H_0 - (H - H_S)/\phi_L$. The straight line in Figure 4 starting at $H_0 = 230$ mm is typical of the constant rate phase of sedimentation. At time approximately equal to 1650 s, the falling rate period is encountered. The first characteristic arising from the origin must be connected to the transition point A between the two periods. As shown in Figure 4, the first characteristic has a slope of approximately $\nu = 8.0\ E(-5)$ m/s.

As a first approximation, we shall assume that the phenomena of characteristics in freely settling suspensions can be transferred to filtration on a horizontal surface, that is, the characteristic and filtrate velocities are additive. The initial filtration rate is in excess of $10\ E(-5)$ m/s, and consequently the characteristic cannot escape the cake. The rate drops from 9.2 to 7.0 $E(-5)$ m/s between 200 and 300 s. Assuming that the characteristics leaving the cake surface have Kynch-type properties, that is, $\nu = d(\phi_s u_s)/d\phi_s$, the first characteristic could escape the cake leaving tangentially at

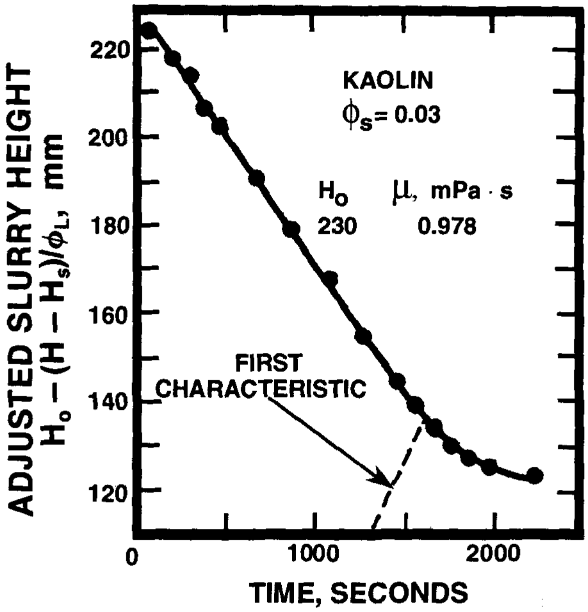

Figure 4. Distance between the clear liquid and settling slurry interfaces is plotted against the time.

about 250 s. The path of the characteristic H_c vs. t is given by an integration of

$$dH_c = (\nu - dv/dt)dt, \tag{22}$$

where $dv/dt = -dH/dt$ and H_c gives the position of the characteristic. Integration yields

$$H_c - L_D = \nu(t - t_D) + H_D - H, \tag{23}$$

where subscript D refers to the position and time at which the characteristic escapes the cake. Curve DE in Figure 3 has been drawn in accordance with Eq. 23. Assuming that DE determines the positions at which the settling velocity is slowed by the rising signal, the slurry to the right of DE will have a higher concentration than 3% by volume. The H_s vs. t curve of Figure 3 can be divided into two sections. The portion AE corresponds to the constant rate period of batch sedimentation, and EC represents the falling rate period. As the slurry concentration in region DEC increases, the rate of deposition due to sedimentation on the cake surface is diminished. The effect on the simplified theory previously presented warrants more investigation.

Parameter Calculations

Conventional analysis

In conventional theory, sedimentation effects are neglected; and Eqs. 10 and 12 take the form

$$\frac{pdt}{\mu dv} = \frac{p}{\mu q} = \alpha_{av} cv + R_m \tag{24}$$

$$\frac{pt}{\mu v} = \frac{p}{\mu q_{av}} = \frac{\alpha_{av}}{2} cv + R_m, \tag{25}$$

where $q_{av} = v/t$ is the average rate. Integration of Eq. 24 assuming p, α_{av}, c, and R_m are constant and inserting the limit $v = 0$ when $t = 0$ leads to Eq. 25. These simplified equations indicate that a plot of the total resistance $R = p/\mu q$ vs. v has the intercept and twice the slope of a plot of the pseudoresistance $p/\mu q_{av}$ vs. v (Tiller, 1990). However, there is a significant difference between the two equations. No assumption has been made concerning the constancy of c or α_{av} in Eq. 24, which represents the instantaneous conditions involving the applied pressure, rate, and cake properties ϵ_{sav} (as reflected in c) and α_{av}. In addition, the medium resistance has not been assumed constant.

Previous experiments run in the author's laboratory have shown that R_m is not constant during the course of a filtration due to migration of fines (Tiller et al., 1981; Leu and Tiller, 1983). As cake builds up and $R_c \gg R_m$, however, the resistance of the medium becomes less important. Properly conducted laboratory experiments should be carried out with media having relatively low resistance if determining α_{av} is the objective.

Although Eq. 25 is deceptively similar to Eq. 24, integration *requires constancy of R_m and the product $c\alpha_{av}$*. As α_{av} can change many fold for a highly compactible material, the parabolic behavior predicted by Eq. 25 is only realized when $R_c \gg R_m$ and $\Delta p_c \to p$. Large deviations from the parabolic form generally occur during the initial stages of filtration (Tiller et al., 1980). If slurries are dilute, that is, ϕ_s/ϵ_{sav} is small, cake buildup will be slow, and the nonparabolic region may extend for a long period of time. For concentrated slurries, the initial period with variable α_{av} may last no more than a few seconds.

In Figure 5, the data of Figure 3 have been plotted in accord with Eqs. 24 and 25 (curve A represents $p/\mu q$ vs. v and curve C represents $p/\mu q_{av}$ vs. v) without regard to the effect of sedimentation. An experimenter who was unaware of the sedimentation problem would assume that the curves of Figure 3 simply represented the volume vs. time discharge data. Examination of curves A and C indicates that they are not linear. Even if they were crudely linearized, the slopes would not be in the ratio of 2/1. Thus the experimenter would be faced with the problem of incompatibility of the experimental data with simplified theory as normally employed.

Theory indicates that tangents drawn to curve A should never have a negative intercept (Tiller et al., 1980). When a $p/\mu q$ vs. v plot is concave upward and yields a negative intercept, an additional resistance over and above that predicted by Eq. 24 appears. Sedimentation and deposition of migrating fine particles are the mechanisms that generally give rise to the additional resistance over and above that predicted by Eq. 24. Curve A exhibits a continuing increase in slope due to sedimentation up to the time (38 min) at which cake formation is complete. At that point, the cake ceases to grow, and the filtrate rate and $p/\mu q$ remain essentially constant (line FB in Figure 3). Curve A undergoes an abrupt change when deposition ceases as evidenced by the horizontal F'B' in Figure 5. We shall return to the constant rate period later.

Although the majority of investigators use $p/\mu q_{av}$ vs. v in analyzing constant pressure filtration, it is the least desirable form. Since an average rate over the entire cycle is used, the response to changes is sluggish. For example, at point F when

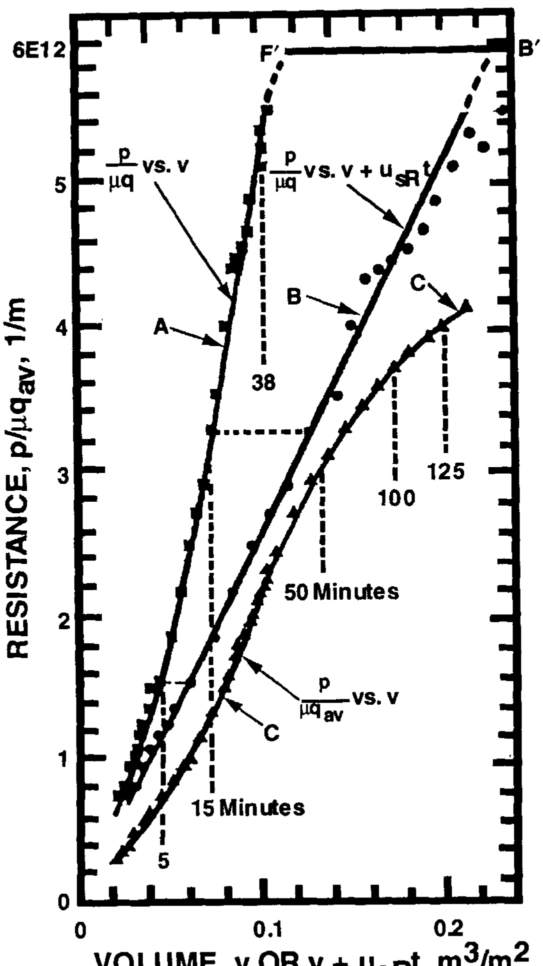

Figure 5. Determination of α_{av} for data in Figure 3.

Conventional plots of $pdt/\mu dv$ (A) and $pt/\mu v$ (C) vs. v yield erroneous results. Plotting $pdt/\mu dv$ vs. $(v + u_{sR}t)$ leads to improved analysis.

$t = 38$ min and cake formation is complete, the $p/\mu q$ plot changes slope rapidly. On the other hand, curve C reaches an inflection point at 38 min, and thereafter the slope begins to decrease. Line F′B′ provides an asymptote that curve C would ultimately approach.

Modified analysis

Accounting for the effect of sedimentation yields an improved analysis of the data. Using Eq. 7 with $u_{sR} = 5.75$ $E(-5)$ m/s leads to curve B in Figure 5. The linearity of B over a wide range lends support to the accuracy of Eq. 19 and its equivalent, Eq. 10.

Although Eq. 19 involving $(v + u_{sR}t)$ provides a better approximation of the physical process, it does not take into account the change in slurry concentration brought about by the appearance of characteristics in the region DEC of Figure 3. Both slurry concentration and the relative settling rate are affected. The solid flux decreases, and consequently, Eq. 19 with a constant value of u_{sR} leads to an overstatement of the value of the deposited solids. Assuming that the straight line drawn for curve B correctly represents the data, it can be seen that points toward the top $(v + u_{sR}t > 0.16)$ fall to the right of the line, indicating that an overestimate was made.

Constant rate period

After all of the solids have been deposited, a constant pressure washing period follows, during which the supernatant liquid flows through the cake. From a simplified viewpoint, since the mass of solids remains constant during the flow of pure liquid and the pressure does not change, the cake resistance should remain constant. As a result, the flow rate would remain constant, and the filtrate volume would be linear in time. This simple model must be carefully evaluated for transient conditions existing during filtration operations.

It is known that the liquid flow rate q varies with distance through the cake as long as solids being deposited at a high porosity at the surface of the cake are compacted as they are covered by new solids (Tiller and Shirato, 1964). As liquid is expressed from a given location, it increases the local flow rate in accord with the continuity equation for the liquid. The local rate of increase of flow rate with respect to the spatial coordinate x is related to the time rate of increase of solidosity by

$$\left(\frac{\partial q}{\partial x}\right)_t = -\left(\frac{\partial \epsilon_s}{\partial t}\right)_x. \qquad (26)$$

As ϵ_s increases with time, $\partial\epsilon_s/\partial t > 0$, and $\partial q/\partial x < 0$. Consequently, q is a maximum at $x = 0$ and decreases to a minimum value at the cake surface where $x = L$. When particle deposition ceases and a new equilibrium structure is attained during the subsequent flow of pure liquid, $\partial\epsilon_s/\partial t = 0$; and q is constant through the cake. Some cake compaction is expected after deposition of solids ceases. The increased compaction leads to higher values of α_{av}.

Textbook definitions of average values of permeability and specific resistance assume that q is constant. Consequently, they are strictly valid only for flow through a bed of constant mass of solids and represent approximations for filtration involving cake growth and compaction. Under most conditions encountered in practice, it is doubtful that q ever varies more than 10–20%. The variation is small for dilute slurries with slow rates of cake buildup.

Comparing calculated values

It is of interest to compare estimates of the specific resistance made on the basis of the curves shown in Figure 5. Because of the changing slopes of curves A and C, estimates of α_{av} vary with the time. Comparisons of calculated values are shown in Table 1. The large differences among the values

Table 1

Method	Average Specific Resistance, m^{-2}		
Filtration	5 min	15 min	38 min
$p/\mu q$ vs. v	10.7 $E(14)$	14.1 $E(14)$	16.1 $E(14)$
$p/\mu q_{av}$ vs. v	10.0 $E(14)$	12.8 $E(14)$	16.6 $E(14)$
$p/\mu q$ vs. $(v + u_{sR}t)$		7.87 $E(14)$	
Washing, flowthrough		8.58 $E(14)$	

is an indication of the errors that may be encountered if sedimentation is neglected. As most laboratory filtrations (including capillary suction testing) take place on horizontal surfaces, accuracy of the reported results must be questioned.

The difference between the values of α_{av} for the plot of $p/\mu q$ vs. $(v + u_{sR}t)$ and the washing or flowthrough period is probably due to the compaction that occurred. Although difficult to measure with precision, the cake compacted from a thickness of 19.5 mm at the end of filtration to 19.0 mm at the end of the flow of clear liquid through the cake. The average solidosity increased from 0.354 to 0.363. The increase of specific resistance of 9% corresponds to a power function relation of the form

$$\alpha_{av} \approx \epsilon_{sav}^{3.4} \tag{27}$$

Bockstal analysis

Integration of Eq. 10 required constancy of α_{av}, ϵ_{sav}, u_{sR}, and R_m. Linearity of the plot of $p/\mu q$ vs. $(v + u_{sR}t)$ in Figure 5 indicates that Eq. 11 should provide a reasonable approximation of the experimental data. Since values of α_{av} (Table 1) differ for the filtering and washing periods, predicted v vs. t results will also differ. A comparison of calculations is shown in Figure 6 where the points represent the experimental data. As expected, the calculated values using α_{av} for the filtration period yield slightly better results. As the average specific resistance during the constant-rate, flowthrough period is larger than the value obtained during filtration, the calculated time is larger than the experimental values. Using a value of α_{av} slightly larger than 7.87 $E(14)$ would yield an even better fit.

Average solidosity

Calculation of the average specific resistance ($\alpha_{av} = R_c/\omega_c$) requires knowledge of the volume of solids per unit area in the cake. Equations 7–9 are based on the constancy of the initial slurry concentration ($\phi_s = 0.03$). The appearance of characteristics in the region CDE of Figure 3, however, affects the calculation of the volume of solids in the cake. Based on Eq. 7, the average solidosity can be obtained as

$$\epsilon_{sav} = \phi_s \frac{H_0 - H_s + L}{L}. \tag{28}$$

Values based on Eq. 28 and shown in Figure 7 were derived from Figure 3 for cake thickness greater than 4 mm and time greater than 300 s. The trend is downward with the lowest value occurring at the point where the slurry disappears, that is, point C in Figures 3 and 7. As the cake builds up, the cake resistance increases as a fraction of the total resistance. The pressure drop across the cake increases, and, as a consequence, the average solidosity should increase. The contrary trend shown in Figure 7 is due to the erroneous assumption of a constant value of ϕ_s.

Estimates can be made of the average suspension concentration by means of a material balance, provided the volume of solids in the cake can be approximated. An increasing slurry concentration in region CD of Figure 2 will account for the erroneously decreasing ϵ_{sav} shown in Figure 7. Letting ϕ_{sav} equal the average concentration and noting that the slurry volume is $(H_s - L)$ leads to

$$\phi_{sav}(H_s - L) = \phi_s H_0 - \epsilon_{sav} L. \tag{29}$$

A value of ϵ_{sav} is required for calculation of ϕ_{sav}. At the end of cake formation (point C in Figure 3), the time was 2279 s, and the average solidosity equaled 0.354. When $t < 2279$ s, ϵ_{sav} would be less than 0.354. Replacing ϵ_{sav} of Eq. 29 by its maximum value of 0.354 leads to an underestimate of the volume of solids and the average volume fraction of solids in the suspension. Calculated values of ϕ_{sav} as shown in Figure

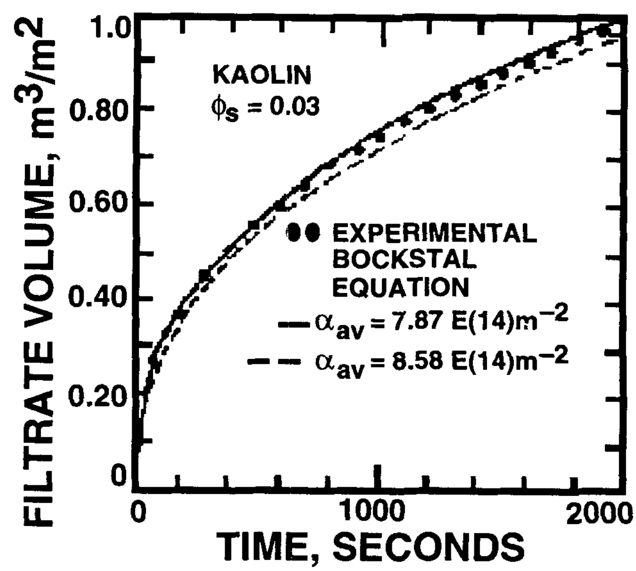

Figure 6. Comparison of experimental v vs. t data with values obtained using the Bockstal equation.

The solid line is based on the filtration period with α_{av} = 7.87 $E(14)$. The broken line is based on the washing period with α_{av} = 8.58 $E(14)$ m^{-2}.

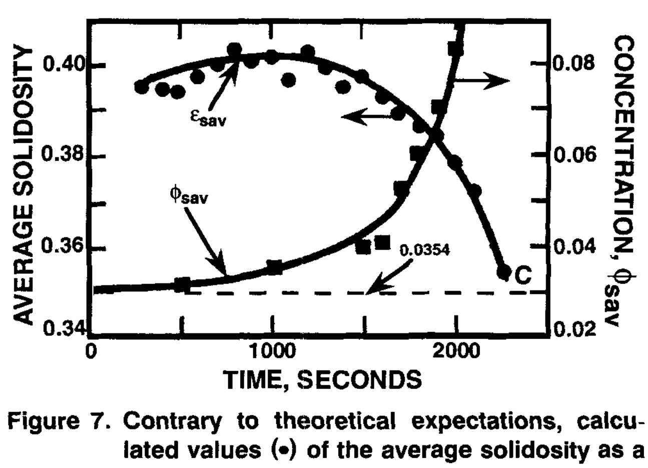

Figure 7. Contrary to theoretical expectations, calculated values (•) of the average solidosity as a function of time show a marked decrease with time.

The curve (■) represents the change in average slurry concentration as based on a material balance.

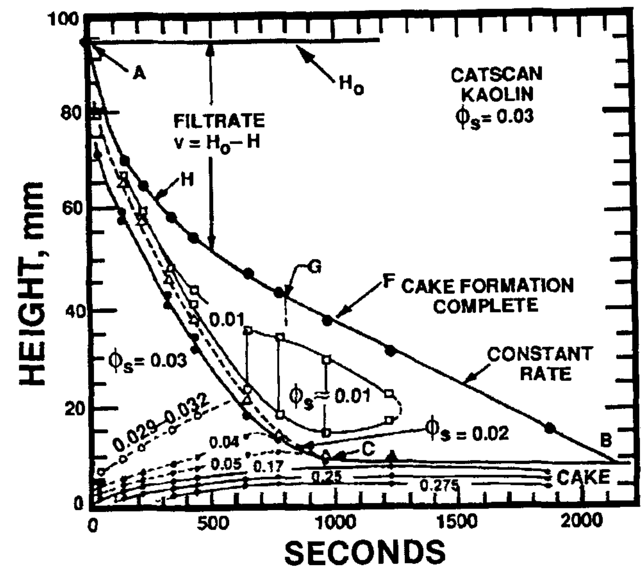

Figure 8. CATSCAN analysis of the pressure filtration of a 3% by volume fraction of solids slurry of kaolin.

Lines of constant average volume fraction of solids are shown. The pressure was 102.1 kPa, and the viscosity was 0.978 mPa·s.

7 increase with time as expected when characteristics are present. The value of ϕ_{sav} begins increasing rapidly around 1500 s, which corresponds to point E in Figure 3. Since the suspension concentration ϕ_{sav} shown in Figure 3 is an average value, the concentration of the slurry just above the cake surface would be higher than the values appearing in Figure 7. We next turn to CATSCAN experiments to verify the existence of variable slurry concentration.

CATSCAN data

Concentration profiles giving the volumetric fraction of solids (ϕ_s in the slurry and ϵ_s in the cake) as a function of time were obtained in a computerized axial tomographic scanner (CATSCAN) for the constant pressure (106.2 kPa, μ = 0.978 mPa·s) filtration of a 3% by volume slurry of kaolin. The data presented in Figure 3 and the CATSCAN data in Figure 8 were obtained on different dates. Since physical properties related to sedimentation of clays are dependent on aging and mechanical treatment, the data would not be expected to be precisely comparable.

Planar scans of the slurry and cake were made from top to bottom of the filter cell at different times. The scan required 3 s and covered a width of approximately 0.5 mm. A blurring of the concentrations occurred at interfaces due to the width of the scan. At each height, the concentration was measured as a function of the horizontal distance. These values were averaged as functions of height and time.

The resulting averaged data for the filtration of 3% by volume slurry of kaolin (flat D) are shown in Figure 8. The CATSCAN reveals details that are not observable with the naked eye. Starting with an initial level of 94 mm, the air/liquid interface is shown as H vs. t. Below the interface, there is a gradual concentration change over 10–25 mm from a zero concentration at the surface (H vs. t) to the full slurry concentration ($\phi_s = 0.03$) along curve AC. The broken line with triangles corresponds to $\phi_s = 0.02$. The $\phi_s = 0.01$ profile is shown as a solid line that evolves into a diffuse region encompassed by squares. The line labeled 0.029–0.032 roughly marks the lower boundary of the constant composition slurry region. What are probably constant composition characteristics are labeled 0.04 and 0.05. The cake surface lies somewhere between the 0.05 and 0.17 curves, which represents a 2–4 mm gap.

The predicted appearance of a characteristic along DE in Figure 3 is supported by the revelation of a region of variable slurry concentration above the cake in Figure 8. The region DEC in Figure 3 with $\phi_s > 0.03$ is comparable to the region in Figure 8 lying between the ϕ_s = 0.029–0.032 boundary and the cake surface.

The CATSCAN data indicate that particles in the zone settling region have varying velocities in contrast to the assumptions underlying the idealized depiction of sedimentation in Figure 2. Turbidity is shown to exist in the supposedly clear region defined by BC in Figure 2 and AECF in Figure 3, and the assumed abrupt concentration jump condition at the cake surface, ϕ_s to ϵ_{s0}, is replaced by a continuous variation of concentration. It is apparent that classical zone settling with a sharply delineated slurry–clear supernatant interface does not exist. Consequently, the equations previously developed in this article must be considered to be approximations. Any theory involving the assumption of zone settling and a discontinuous jump in concentration between the slurry and cake should be reevaluated.

Gravitational Filtration Experiments

The effect of sedimentation is magnified in filtrations where gravity is the driving force. The magnitude of the gravitational contribution can be judged with the assistance of Eq. 19, which employs the term $(v + u_{sR}t)$. At a given time, the $u_{sR}t$ term has a constant value, whereas v depends upon the applied pressure. Increasing pressure leads to increased filtrate rates and larger values of v. When very low gravitational heads are employed, the $u_{sR}t$ term would be expected to dominate. The results of a gravitational filtration experiment of a slurry with 3% by volume of kaolin are shown in Figure 9. Starting with an initial head of H_0 = 48 cm (point A), a plot of the visually determined liquid/gas interface height H, the slurry/liquid interface H_s, and the cake height are shown as functions of time up to 8000 s.

The relative settling velocity of 5.84 $E(-5)$ m/s checks closely with the values determined during pressure filtration. The average filtrate velocity of 6.6 $E(-6)$ m/s over the first 5000 s was only 12% of the relative settling velocity. At 5000 s, the upper liquid level had fallen from 48 to 43.7 cm, while the slurry/supernatant liquid level has dropped to 14.5 cm. A rough estimate indicates that 75% of the solids were in the cake at 5000 s, while only 7% of the filtrate had passed through the cake. Clearly, sedimentation dominated the process.

The first characteristic for pure sedimentation without filtration was estimated to have a path given by OB in Figure 9. The value of the characteristic velocity along OB is approximated as 4.25 $E(-5)$ m/s, which does not compare favorably

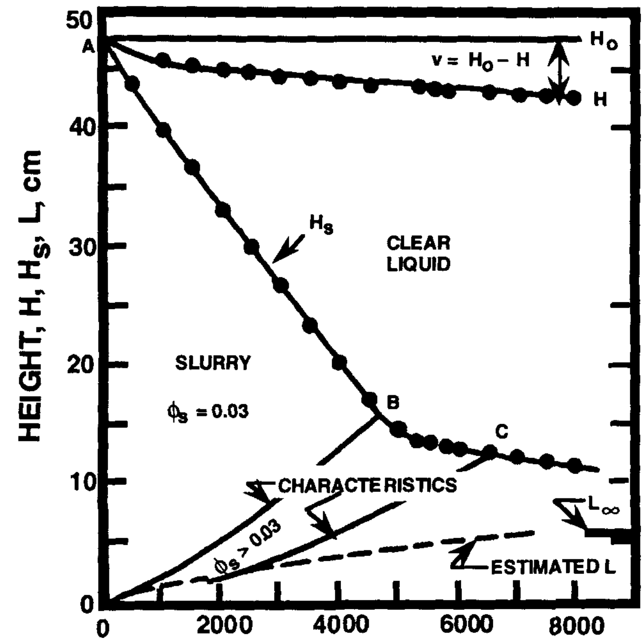

Figure 9. Heights of liquid, suspension, and cake as a function of time in gravitational filtration of a 3% by volume slurry of kaolin.

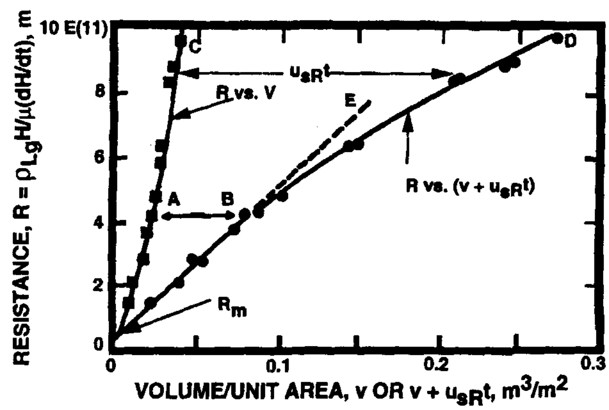

Figure 10. Data analysis involving plots of cake resistance vs. v and $(v+u_{sR}t)$.

with the estimate of 8.0 $E(-5)$ m/s for the data of Figure 3. The actual path OB of the characteristics with filtration was obtained by subtracting the distance $(H_0 - H)$ traversed by the upper level from the path of the first characteristic given by $H = 4.25\ E(-5)t$. A similar construction was carried out along OC. Slurry concentrations lying above OB should equal the initial value of $\phi_s = 0.03$. Concentrations of slurries between OC and the cake surface are expected to be greater than 0.03.

In Figure 10, a plot of the resistance $R = pdt/\mu dv$ is shown as a function of v and $(v + u_{sR}t)$. In contrast to the previous analysis in Figure 3, the pressure is not constant, and the resistance is given by

$$R = -\frac{\rho_L gH}{\mu dH/dt}. \tag{30}$$

The minus sign is required because dH/dt is negative. At $v = 0$, the two plots intersect the resistance axis at R_m. Thereafter they differ by $u_{sR}t$.

Points A and B represent the period for which t is less than 1000 s. Up to that time, the effect of the concentration change of the slurry due to the rising characteristic along OC in Figure 9 is small. Past 1000 s, the relative sedimentation velocity u_{sR} decreases, and $u_{sR}t$ overstates the incremental quantity of solids deposited on the cake. As a consequence, the slope of the line decreases beginning about point B. Up to B, the basic postulate that α_{av} and u_{sR} are constant is supported by the linearity of the line. Past point B, u_{sR} decreases, and use of the initial value of 5.84 $E(-5)$ m/s leads to an overestimate of $(v + u_{sR}t)$ equal to the difference between the dashed and solid lines.

With R_m having an estimated value of 3.0 $E(10)$ m^{-1}, the average specific resistance based on the $(v + u_{sR}t)$ curve was found to be $\alpha_{av} = 1.36\ E(14)\ m^{-2}$. Calculations based on the $p/\mu q$ vs. v data up to point A yielded $\alpha_{av} = 5.1\ E(14)\ m^{-2}$. Thus failure to account for sedimentation produced a value 3.75 times larger than the α_{av} arising from the $(v + u_{sR}t)$ curve. In general, parameter calculations based upon data arising from filtrations depending upon gravity for a driving force would be subject to unacceptable errors if sedimentation were neglected.

A comparison of the values of α_{av} obtained in the gravitational and pressure filtrations can be made by assuming that the specific resistance is related to a power function of the pressure. In the gravitational filtration, the height of the liquid dropped from 48 to 46 cm during the first 1000 s. The average hydrostatic pressure during that period was 4.61 kPa. Using the values of 104 kPa and $\alpha_{av} = 7.87\ E(14)\ m^{-2}$ in Table 1, the following relation results:

$$\alpha_{av} = 5.75\ E(13) p^{0.563} \tag{31}$$

The exponent is reasonable for kaolin.

Conclusions

Sedimentation plays a major role in batch filtrations on horizontal surfaces. Material balances based on the filtrate volume alone without corrections for the sedimentation effect can lead to substantial errors and a gross overestimate of the average specific resistance. Errors are magnified where gravitational driving forces and dense particles are involved.

CATSCAN analyses of filtration and sedimentation operations indicate that the sharp discontinuities assumed to exist in zone settling and cake formation do not exist. Further theoretical analysis is needed to clarify the behavior of conventional zone settling processes.

The appearance of characteristics in filtration leads to changes in slurry concentration, further complicating the material balances that are based on the assumption of constant suspension concentration.

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Notation

c = volume of solids/unit volume of filtrate
g = acceleration due to gravity, m/s^2
H_D = value of H_c at cake surface, m
L = cake thickness, m
L_D = value of L when $t = t_D$, m
L_f = value of L when slurry disappears, m
p_0 = value of p at $t = 0$, Pa
t_D = time at point D, Figure 3, s
t_f = time at which slurry disappears, s
v_f = value of v at time t_f, m^3/m^2

Greek letters

ν = characteristic velocity, m/s
ρ_L = liquid density, kgm/m^3
ρ_s = solid density, kgm/m^3
ϕ_s = volume fraction of liquid in slurry
ϕ_{sav} = average value of ϕ_s
ω = volume of dry solids/unit area between the medium and any position in the cake, m^3/m^2

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